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Exam II: MTH 213, Fall 2018

Ayman Badawi

Score = $\frac{52}{52}$ excellent!

→ undetermined.

QUESTION 2. (6 points)) Consider the following code

For $k = 3$ to $(n^2 + 4)$ → $n^2+4-3+1 = (n^2+2)$ times

$S = k^2 + 3 * k + 4$ → 4 times

For $i = 2$ to $(k + 7)$ → $k+7-2+1 = (k+6)$ times

$L = i^2 + 7 * i + 2$ → 4 times

next i

next k

(i) Find the exact number of addition, subtraction, multiplication that the code executed.

\# of executions of outer loop $= (n^2+2)$ times

outerloop $k=$	$K=3$	$K=n^2+4$
innerloop $i=$	# of executions $= 9$ times # of operations $= 4\times 9 = 36$	# of executions $= n^2+4+6 = (n^2+10)$ times # of operations $= 4\times(n^2+10) = 4n^2+40$

→ # of operations of outerloop $= 4(n^2+2) = 4n^2+8$

∴ the exact # $= \frac{(36+4n^2+40)(n^2+2)}{2} + 4n^2+8$.

(ii) Find the complexity of the code.

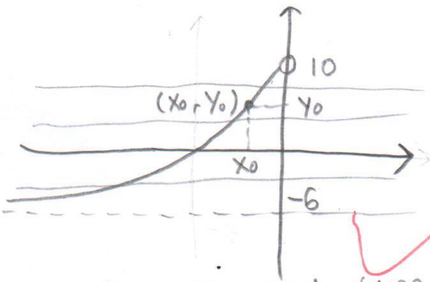
$O(\text{code}) = n^4$

 DaHyun Kim

QUESTION 3. (5 points)) Show that $|(-6, 10]| = |(-\infty, 0)|$.

$f(x) = 16e^x - 6$

$f: \overset{\text{domain}}{(-\infty, 0)} \longrightarrow \overset{\text{co-domain}}{(-6, 10)}$



By looking at the graph, we can know that f(x) is bijective. Since a bijective function can be constructed, the cardinality of the domain is equal to the cardinality of the co-domain.

Also I need this result: Assume $|A| = \infty$, B is countable.
Then: $|A \cup B| = |A|$
$|(-6, 10)| = \infty$, $\{10\}$ is countable. $\Rightarrow |(-6,10) \cup \{10\}| = |(-6, 10]| = |(-6, 10)|$
Since cardinality is transitive, $|(-6,10)| = |(-6,10]| = |(-\infty, 0)|$

QUESTION 4. (6 points)) (a) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 3 & 6 & 7 & 5 \end{pmatrix}$

Find the least positive integer n such that $f^n = I$.

cycle $\Rightarrow$ $(1\,2)(3\,4)(5\,6\,7)$

LCM $[2, 2, 3] = 6$ $\therefore$ the least positive integer n is 6.

(b) Clearly, f is invertible. Find f^{-1}.

$f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 3 & 7 & 5 & 6 \end{pmatrix}$

QUESTION 5. (6 points)) Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $H = \{0, 4\}$. Define "=" on A such that $\forall\, a, b \in A$, we have "a = b" if $(a - b)\ (mod\ 8) \in H$. Then "=" is an equivalence relation (Do not show that). Find all equivalence classes. Is $(2, 7) \in$ " = "? How many elements does "=" have? (view "=" as a subset of $A \times A$)

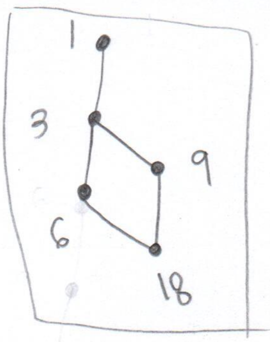
a) $[0] = \{0, 4\}$
$[1] = \{1, 5\}$
$[2] = \{2, 6\}$
$[3] = \{3, 7\}$

b) $(2, 7) \notin$ "="
Since $2 \text{"}\neq\text{"} 7$, They are not in the same equivalence class.

c) $2^2 + 2^2 + 2^2 + 2^2 = \boxed{16}$

QUESTION 6. (6 points)) Let $A = \{1, 3, 6, 9, 18\}$. Define "<" on A such that $a < b$ iff $a = bc$ for some $c \in \{1, 2, 3, 6, 9, 18\}$. Then "<" is a partial order on A (do not show that). Draw the Hasse diagram of such relation.

1 "<" (1)
3 "<" (1, 3)
6 "<" (1, 3, 6)
9 "<" (1, 3, 9)
18 "<" (1, 3, 6, 9, 18)



i) Find $9 \vee 1$ 1

ii) Find $9 \wedge 1$ 9

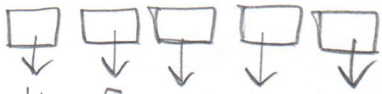
iii) Find $9 \wedge 6$ 18

iv) Find $9 \vee 6$ 3

If there is a least element? what is it? yes, 18

If there is a greatest element? what is it? yes, 1

QUESTION 7. (3 points)) How many 5-digit odd integers greater than 20000 can be formed using the digits $\{1, 2, 5, 7, 8\}$



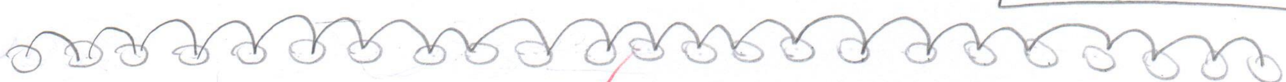
> 20000

4 5 5 5 3

$4 \times 5 \times 5 \times 5 \times 3 = 1500$

QUESTION 8. (3 points)) 23 females are standing in a row and each female shakes hands only with the female on her left and with the female on her right exactly once. How many handshakes took place? (note that female number 1 receives only one shake hand and so is female number 23).

$\boxed{22 \text{ handshakes}}$



$23 - 1 = 22$

QUESTION 9. (5 points)) 326 persons are in a gathering. We know that each person was born on Tuesday or Thursday or Saturday and each person was born in March or May or June or October or December. Then there exist at least n persons who were born in the same month and at the same day. Find the maximum value of n.

Domain : 326

Co-domain : $3 \times 5 = 15$

$|326| > |15|$

Since the cardinality of the co-domain is less than the cardinality of the domain, we can use the pegeonhole method.

$n = \left\lceil \frac{326}{15} \right\rceil = 22$ ∴ the maximum value of n is 22.

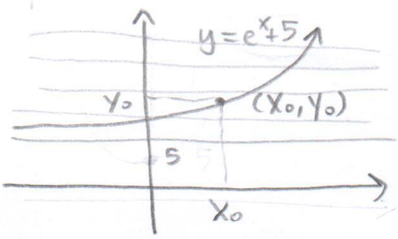
QUESTION 10. (6 points)) (1) Imagine that you just heard someone is saying that the set A is countable? What does that mean?

If set A is countable that means the set has finitely many elements in the set or there exists a bijective function from A to N^*.

(2) Is $|(5, \infty)| = |R|$ if yes, then construct a bijective function between them.

$f(x) = e^x + 5$

$f : \underset{\text{domain}}{(-\infty, +\infty)} \rightarrow \underset{\text{co-domain}}{(5, \infty)}$



By looking at the graph, we can know that f(x) is bijective. Since we constructed a bijective function, the cardinality of the domain is equal to the cardinality of the co-domain.

$\boxed{f(x) = e^x + 5}$

Faculty information

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